$$I(x;y) = H(Y) - H(Y/x)$$
$$= 0.97095 - 0.12451$$
$$= 0.12451.\ \text{bits}.$$

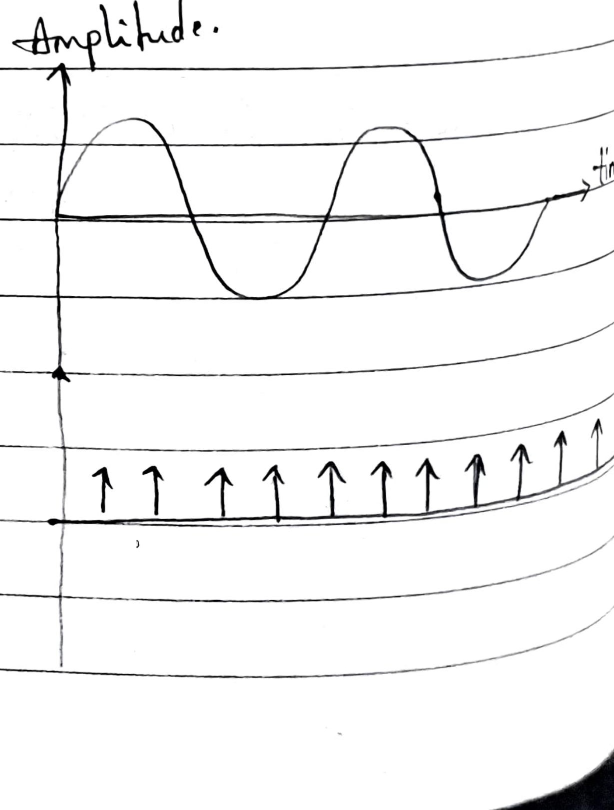
The unstertinity about y about .12451

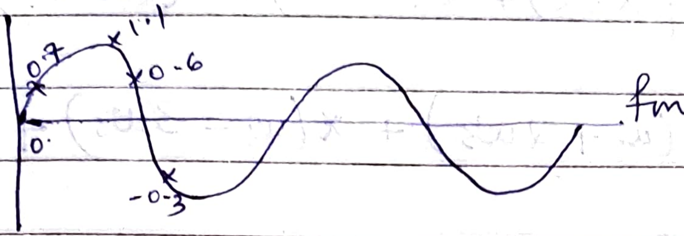
Module - 3.

PCM - Pulse Code Modulation.
DPCM - Differentiated Pulse Code Modulation..
Delta Modulation.

Analog Signal.

1. Sampling.
2. Quantization.
3. Coding.
4. ~~Digital~~.





Sampling Sgnl.

$f_N = f_s = 2f_m$.

Sampling Theorem.

A continous - time signal may be completely respresented in its samples & recovered back if the sampling freq is $f_s \geq f_m$.

$$g(t) = x(t)\,\delta_{T_s}(t) \longrightarrow (1)$$

$$\delta_{T_s}(t) = \frac{1}{T_s}\left[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + 2\cos 3\omega_s t + \dots\right] \longrightarrow (2).$$

$\omega_s = 2\pi f_s$

Sub (2) in (1).

$$g(t) = \frac{1}{T_s}\left[x(t) + 2x(t)\cos\omega_s t + 2x(t)\cos 2\omega_s t + 2x(t)\, 3\omega_s t + \dots\right]$$

$$G(\omega) = \frac{1}{T_s}\Big[X(\omega) + X(\omega-\omega_s) + X(\omega+\omega_s) + X(\omega-2\omega_s)$$

$$+ X(\omega+2\omega_s) + X(\omega-3\omega_s) + X(\omega+3\omega_s) + \ldots$$

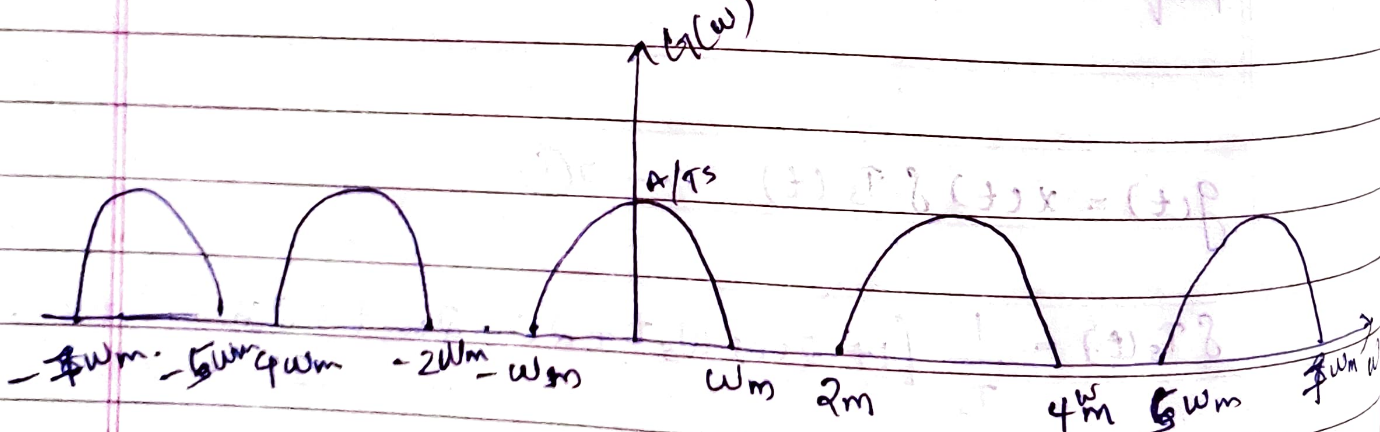
$$G(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$$

<u>CASE - 1</u> $[\omega_s > 2\omega_m]$

Eg:- $\omega_s = 3\omega_m$.

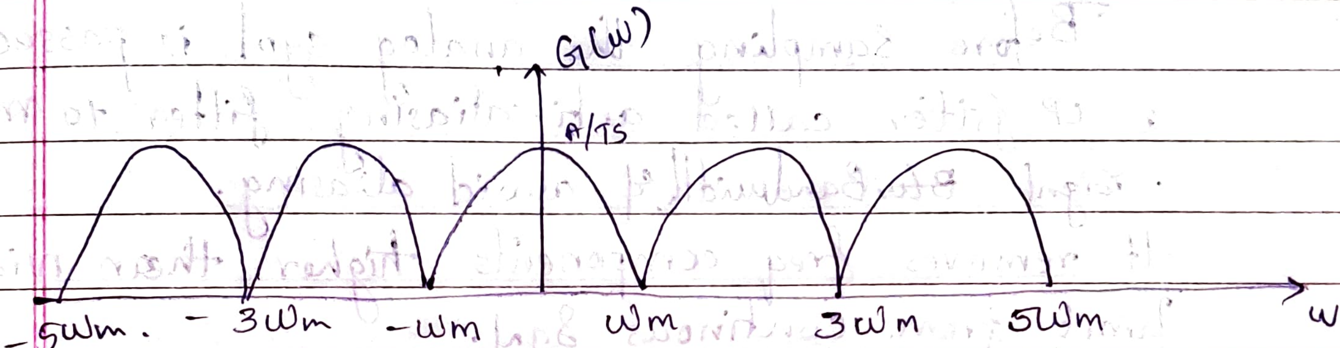
$$G(\omega) = A/T_s\Big[X(\omega) + X(\omega-3\omega_m) + X(\omega+3\omega_m) +$$

$$X(\omega-6\omega_m + X(\omega-9\omega_m) + \ldots\Big]$$



CASE - 2 $[\omega_s = 2\omega_m]$

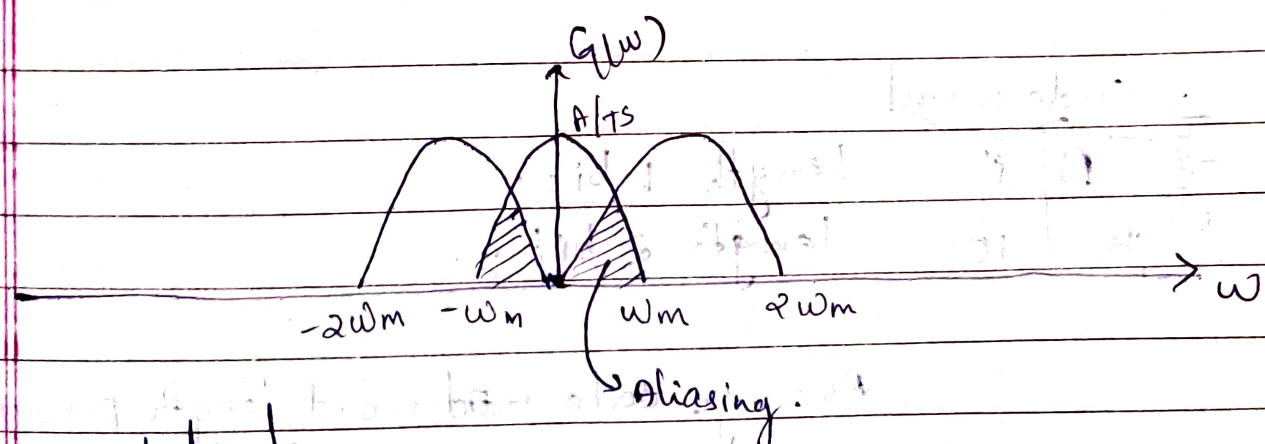
$$G(\omega) = \frac{1}{T_s}\Big[X(\omega) + X(\omega - 2\omega_m) + X(\omega + 2\omega_m) + X(\omega - 2\omega_m) + X(\omega + 4\omega_m) + X(\cdots)\Big]$$



CASE - 3 $[\omega_s < 2\omega_m]$

$(\omega_s = \omega_m)$

$$G(\omega) = \frac{1}{T_s}\Big[X(\omega) + X(\omega - \omega_m) + X(\omega + \omega_m) + X(\omega - 2\omega_m) + X(\omega + 2\omega_m) - - - - - -\Big]$$



Sampled sgnl. $\longrightarrow$ $\boxed{\text{LP filter.}}$ $\longrightarrow$ msg sgnl.

Reconstruction filter.

Page No.
Date

→ Aliasing is the overlaping of adjuscent components when the sampling freq $f_s < 2f_m$.

PRE Filtering [Anti-Aliasing filter]

Before sampling the analog sgnl is passed through a LP filter called anti-aliasing filter to restrict . Signl ~~B/w~~ Bandwidth & avoid aliasing.
It removes freq components higher than Niquist limit from continous sgnl.

Source Coding

Source → Symbols { A, B, C ... / 1, 2, 3 ...

Code word

Symbol:

A	1	length 1 bit.
B	10	length 2 bit.

Average code ~~word~~ word length per Symbol

$L = 0.75 \times 1 + 0.25 \times 2$
$= 1.25$ bits/Symbol.

Page No.

Date

→ Aliasing is the overlaping of adjuscent components when the sampling freq $f_s < 2f_m$.

PRE Filtering [Anti-Aliasing filter]

Before sampling the analog sgnl is passed through a LP filter called anti-aliasing filter to restrict signl ~~B/w~~ Bandwidth & avoid aliasing.
It removes freq components, higher than Niquist limit from continous sgnl.

Source Coding

Source → Symbols $\begin{cases} A, B, C \ldots \\ 1, 2, 3 \ldots \end{cases}$

Symbol · Code word

Symbol	Code word	
A	1	length 1 bit.
B	10	length 2 bit.

Average code ~~word~~ word length per Symbol

$L = 0.75 \times 1 + 0.25 \times 2$
$= 1.25$ bits/Symbol.

Page No.
Date

→ In source coding each source symbol [like a letter, digit or sgnl sample] is assigned a code word [a seq of bits] Some symbols may have shorter code words [if they occur often] & some may have longer code words [if they occur rarely] if the source symbols are $\{x_1, x_2, \ldots x_n\}$

→ each symbol x_i has probability P_i

→ Each symbol is assigned a code length l_i (in bits)

→ Then avg code word length per symbol

$$\boxed{L = \sum_{i=1}^{n} P_i l_i}$$

→ Entropy $H(x)$ is the min possible avg no: of bits per symbol needed to represent a source.

3M ## Shanon's Source Coding Theorm

For a discrete memory less source with entropy $H(x)$, it is possible to encode the o/p of the source into binary digits such that the avg. code word length per symbol L satisfy

$$H(x) \leq L \leq H(x) + 1$$

ie, No Source can be encoded with an avg. code word length ~~th~~ less than its entropy.

Source Coding Theorm II
Shanon's Noiseless

For a discrete memory less source with entropy H(s) if the source code in encoded with rate $R \geq H(x)$ bit/symbol. Then it is posible to encode & decode with arbitarilly small probab ~~with~~ error by choosing sufficiently long code words [large block length]
In short entropy H(x) is the min compression rate achievable without loss of infor.

Pulse Code Modulation. PCM

Analog i/p Sgnl. → LPF → Sampler → Quantization → Encoder →

It is a method to digitaly represent digital Signal.
• It is the most common technique for converting an analog Signal (like, Voice, music, sensor data) into

digital form, so that it can be processed, stored or transmitted by digital systems.
The block diagram of PCM transmitter is shown in fig.

1. Low Pass Filter.
- Removes freq. above the max msg sgnl freq. 'f_m.'
- Prevents aliasing during sampling.
-

2. Sampler
- Takes discrete values (samples) from the continous time analog sgnl at uniform time intervals.
- Sampling convert the continous time i/p sgnl into discrete time sgnl.
- To avoid loss of information, the sampling freq 'f_s' $f_s \geq 2f_m$
where f_m – max freq. of continous time sgnl.

3. Quantizer
- Each sampled value is mapped to nearest quantisation level.
- The range of the sgnl is divided into L levels. (uniform or Non-uniform)
- Each Sample is rounded (approximated)
- Introducing a quantisation error (Quantisation noise)

- If we use 'n' bits per sample, then no. of quantisation level $L = 2^n$
  Eg :- 8 bits = 256 (2^8) quantisation

4. Encoder

- each quantised level is assigned a binary code These binary no. form the PCM digital sgnl.
- The no. of bits per sample (n) decides the resolution (fidelity of reconstruction) & bit rate

5. Transmission

The stream of binary codes is transmitted as a seq of electric pulse, Light pulse (fiber optics) or radio waves.

PCM - Reciever

PCM Sgnl → Decoder → Reconstruction filter → Analog Sgnl (Reconstructed Sgnl)

1. Decoding.

Binary Code ⟶ Quantization Signal Levels.

2. Reconstruction

It is a LPF that smoothens the staircase like quantised sgnl back to a continous time analog wave form.

Bit Rate of PCM

The total bitrate required for PCM is

$$R_b = n \times f_s$$

$n \rightarrow$ no. of bits / sample

$f_s \rightarrow$ Sampling freq

Eg :- For voice (bandwidth = 4 KHz)

$f_s = 8$ KHz (8000 Hz)

$n = 8$ bit / sample.

$R_b = 8 \times 8000 = 64000$ bit per sec

$= 64$ k bps

Page No.
Date

Advantages of PCM.

1. High Noise Immunity.
2. Easy to store, process & Multiplex
3. ~~Process~~ Compactable with digital Systems & Encryption
4. Standardized [used in telephone, CD, DVD, digital audio]

Disadvantages of PCM

1. Require higher bandwidth compare to analog transmision
2. More complex circuitery.
3. Quantization introduce distortion/noise.

Applications

1. Digital Telephone [standard 64 kbps voice channels]
2. Audio CD, DVDs, blueray disc
3. Satelight & Space Communication.
4. Video & image transmision.
5. Data transmision in computer network.

Page No.
Date / /

1\. A PCM system uses a uniform quantizer followed by an 8 bit encoder. If the bitrate of the system is 108 bps. Then what is the max bandwidth of the lowpass msg sgnl for which the systm operates satisfactorly.

$R_b = 108$ bps $\quad R_b = n f_s$, $n = 8$ bits.

$= 108 = 8 \times f_s$

$f_s = \frac{108}{8} = 13.5$ samples/sec.

Nyquist Criteria

$f_s \geq 2 f_m$

$f_m \leq \frac{f_s}{2}$

for satisfactory operation the max msg sgnl bandwidth must be $\leq$ half of the sampling rate.

$f_m \leq \frac{13.5}{2} \leq \underline{6.75 \text{ Hz}}$.

$\therefore$ The max bandwidth of LP msg sgnl is 6.75 Hz.

Page No.
Date

Waveform Coding.

Continous time sgnl $\longrightarrow$ Digital.
(Analog)

It is a source coding technique used in digital Communication system, where the goal is to represent a continous time sgnl (waveform) in digital form, So that it can be efficiently transmitted or stored.

Instead of transmitting the original analog sgnl, waveform coding samples it, quantizes the samples into discrete values & then encodes them to binary form.

The process aim to preserve the essential shape (waveform) of the sgnl, by dreducing the amount of data.

3 Main Steps in Waveform Coding :-

1) Sampling
2) Quantization
3) Encoding.

Quantization is the process of approximating a continous amplitude sgnl into discrete amplitude sgnl.

Page No.
Date / /

→ It is a key step PCM after Sampling.

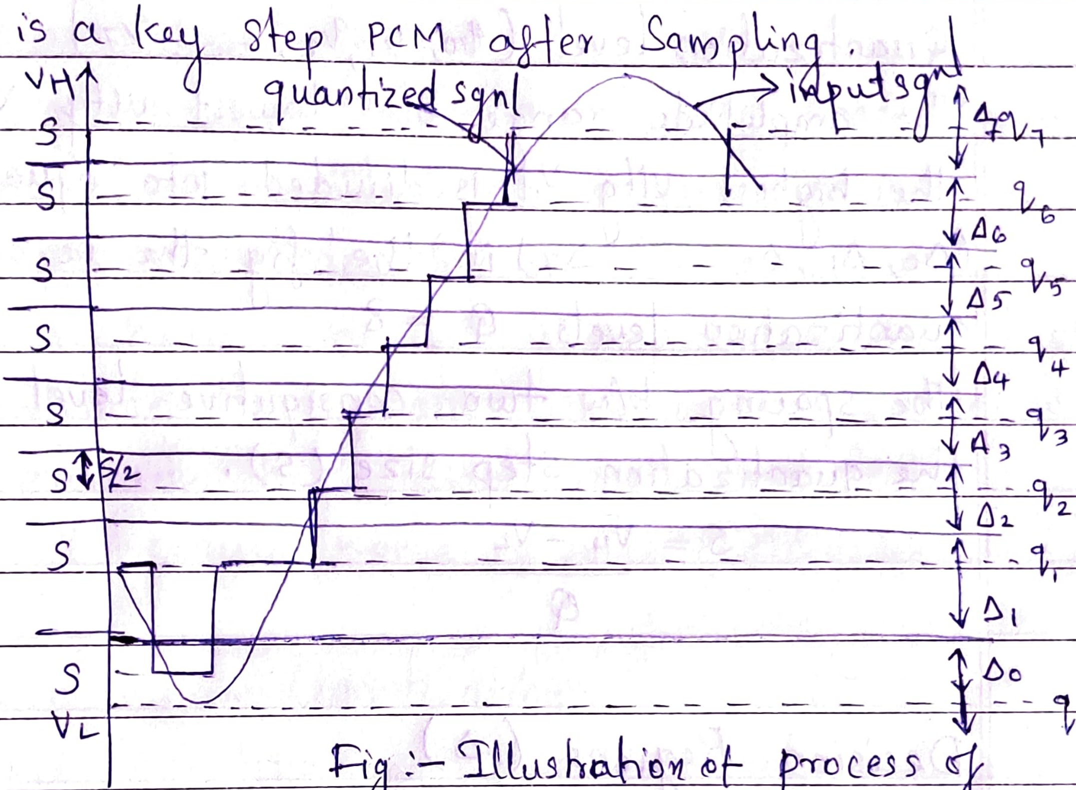


Fig :- Illustration of process of quantization.

$$S = \frac{V_H - V_L}{\text{No. of quantisation level}}$$

i/p sgnl $x(t)$ is the continous time, continous amplitude sgnl, it varies smoothly with time.

Quantized sgnl $x_q(t)$

The i/p sgnl $x(t)$ is approximated to the nearest quantization level shown as horizontal steps (in the fig). instead of +ve amplitude value, the ~~instre~~ sgnl now takes only a finite set of levels $q_0, q_1, q_2 \dots q_7$

Page No.

Date

Quantization level ($q_0, q_1, q_2, \dots q_7$)
The amplitude range b/w lowest vltg V_L & the highest vltg V_H is divided into equal intervals ($\Delta_0, \Delta_1, \Delta_2 \dots \Delta_7$) in the fig the no. of quantization levels $Q = 8$
The spacing b/w two consiqutive level is called the quantization step size (S).

$$S = \frac{V_H - V_L}{Q}$$

Decision Regions (Δ)
Each interval ($\Delta_0, \Delta_1, \dots \Delta_7$) is a region where any i/p value gets mapped into the same quantization level.
eg:- If $x(t)$ falls inside Δ_3 it is rounded into level q_3

Quantization Noise (Error)
The shaded diff betwen the original Sgnl $x(t)$ & quantized version $x_q(t)$ is the quantization error. This is the error due to rounding.
Smaller step size s meets lesser error.
Increasing the no. of levels Q reduces this error but require more bits for coding

Page No.
Date / /

Uniform Quantization

The quantization levels are equally spaced across the entire amplitude range. Step size is same for all i/p amplitudes for signals like speach or audio which have ~~to~~ more low amp. values than high amplitude ~~values~~, ones, uniform quantisation ~~It~~ produces high quantization error for weak sgnl (because noise is more noticeable there)

Non Uniform Quantization.

Here quantization values are not equally spaced, Small step size for low amplitude (to reduce quantization eror) & large step size or high amplitudes (where noise is less noticeble).

Companding.

Companding = Compressing + Expanding.

~~At transmitt~~ At the Transmitter, the amplitude of the i/p signal is compressed [large amp. are reduced more than small ampltides]. At the recieve the sgnl is expanded back to its original form

Page No.
Date

Significants of Comparators.

→ Improve sgnl to quantisation ~~to~~ noise ratio.
→ In uniform quantisation low amplitudes suffer from high quantisation noise–companding reduces this effect by allocating more quantisation levels to weaker sgnls & fewer to stronger levels.
→ Efficient use of bits.
→ Reduces Quantisation error. for low level sgnls.
→ Maintains dinamic range.

$$\text{dynamic range} = \frac{\text{max possible Sgnl amp}}{\text{min detectable Sgnl amp}}$$

it indicates how well a systm can handle both very weak & very strong sgnls

Common Companding law

1). μ-law : used in Northamerica & Japan.
2). A_law : used in Europe & India

Page No.
Date / /

μ- Law Companding (mμ-law) Nonuniform Quantization techn.

→ It is a companding technique used to mainly in PCM systm. It compresses the dynamic range of anolog sgnls before quantization & transmission. & then expands them back āt reciever.

→ The goal is to improve the sgnl to quantisation noise ratio, especially for weak sgnls like speach

Compression law (at transmitter)

The i/p sgnl amplitude x is compressed using the μ-law formula.

$$\boxed{y = \frac{\ln\left(1 + \mu \frac{|x|}{x_{max}}\right)}{\ln(1+\mu)} \cdot \text{Sgn}(x)}$$

x → i/p Sgnl amplitude.

x_{max} → max amplitude of i/p sgnl.

y → Compressed Sgnl

μ → Compression parameter ($\mu = 255$,)

Sgn(x) → Presserves the sign of i/p (+/-)

Page No.
Date / /

Expansion Law (at reciever)

At the reciever the compressed sgnl is expanded back using the formula

$$x = \mathrm{Sgn}(y) \cdot X_{max} \cdot \frac{(1+\mu)^{|y|} - 1}{\mu}$$

(1) For a non-uniform μ-law quantizer compute quantized value of a sgnl that is normalised to 0.75 with $\mu = 255$

Let, $X_{max} = 1$

$x = 0.75$

$\mu = 255$

$\mathrm{Sgn}(x) = +1$, Since i/p is +ve.

$y = ?$

$$y = \frac{\ln\left(1 + \mu \cdot \frac{|x|}{X_{max}}\right)}{\ln(1+\mu)} \cdot \mathrm{Sgn}(x)$$

$$= \frac{\ln\left(1 + \overset{\cancel{0.875}}{255 \times 0.75}\right)}{\ln(1+255)} \times 1$$

$$= \frac{5.257}{5.545} = \underline{0.948}$$

Page No.
Date / /

for a normalized i/p $x = 0.75$, $\mu = 255$ the μ-law quantized value is 0.948.

② Expand the compressed value back & show it recovers the original.

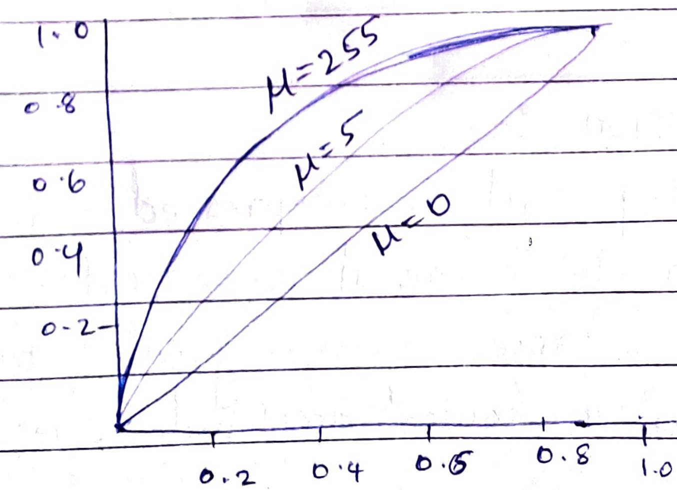
$$x = \mathrm{Sgn}(y) \cdot X_{max} \cdot \frac{(1+\mu)^{|y|} - 1}{\mu}$$

$$= 1 \times 1 \times \frac{(1+255)^{0.948} - 1}{255}$$

$$= \frac{256^{0.948} - 1}{255} \Rightarrow \frac{192.25 - 1}{255}$$

$$= \frac{191.25}{255} = \underline{\underline{0.75}}$$

μ-law Compressor characteristics.



Page No.
Date

→ ~~It is used in~~ pulse code modulation.

Application :-

→ µlaw companding is a non-uniform quantisation technique that applies logarathmic compression before quantisation & expansion after reception, gaining better SNR (Sgnl to noise for low amplitude sgnl & efficient use of quantisation levels. Larger the value of µ, results in larger Compression of o/p to i/p with higher amplitudes. It is widely used in digital telephone (PCM system, G711 std).

Recent digital syst uses 8 bit PCM with $\mu = 255$

A-law Companding

It is a form of companding, involves both compression & expansion.

Compression :-

The analog sgnl is compressed using a non-linear transfer fn before it is converted into a digital sgnl. This process gives a greater propotional emphasis to lower amplitude sgnls. The formula for

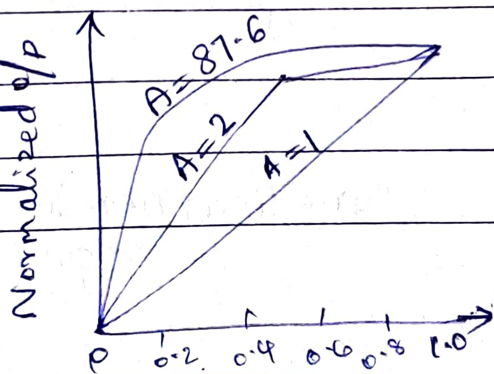
The formula for A-law compression for normalised i/p 'x' is

$$y = \begin{cases} \dfrac{A|x|}{1+\ln(A)}, & \text{if } 0 \le |x| \le \dfrac{1}{A} \\[2ex] \dfrac{1+\ln(A|x|)}{1+\ln(A)}, & \text{if } \dfrac{1}{A} < |x| \le 1 \end{cases}$$

where, A = Compression parameter.
its typical value is 87.6

Expansion :-
At the recieving end an inverse fn is applied to the digital sgnl to expand it back to its original dynamic range restoring the sgnl with minimal loss of information.

A-law Compressor characterstics.



normalized i/p

Page No.
Date

Application :-

A-law companding is essential for pulse code Modulation System in digital telephony. It allows for the efficient encoding of a wide dynamic range of speech sgnls into a smaller no. of bits, typically converting a 12 or 13-bit linear sgnl to an 8-bit non-linear dgtl sgnl. This reduction in bit depth is a key reason for the widespreded use of 64 Kbps in digital voice communication, which is a standard data range for a single voice channel.

Features	A-law	μ-law.
Region of use	Europe & rest of the world	North america & Japan
Compression parameter	A = 87.6	μ = 255
Primary Difference	Slightly better sgnl to noise ratio than μ-law for higher amplitude sgnls	Slightly larger dynamic range than A-law at the expense of a lower SNR for Small sgnls.
Origin Behavior	Curve linear near the origin	Curve is logarithmic near the origin.

Page No.

Date

① Determine the stepsize (Δ) of a DM if the i/p is $10 \cos(2000\pi t)$ & Sampling size is 50000 samples per sec.

Ans – To find Δ ?

Condition to avoid slope overload distortion

$$\left|\frac{dx(t)}{dt}\right|_{max} \le \frac{\Delta}{T_s} \le \Delta f_s \longrightarrow ①$$

given, $x(t) = 10 \cos(2000\pi t)$

$f_s = 50000$

$$\frac{dx(t)}{dt} = 10 \times (-2000\pi) \sin(2000\pi t)$$

$$= -20000\pi \sin(2000\pi t)$$

$$\left|\frac{dx(t)}{dt}\right|_{max} = 20000\pi$$

Sub it in eqn ①

$$20000\pi \le \Delta f_s$$

$$\frac{20000\pi}{f_s} \le \Delta$$

$$\frac{20000}{50000} = 1.256\ \le \Delta$$

$\Delta \ge$ ~~12.56~~ 1.256 V

min stepsize must be ~~12.56~~ 1.256 V to avoid slope overload distortion.

Page No.
Date

(2) The i/p to the DM is a sinsoidal sgnl whose freq varies from 500 to 5000 Hz, Sampling rate is 4 times the nyquist rate. The sgnl peak amplitude is 1V. Determine the step size. when the Sampling freq is 1000 Hz

To avoid slp overload distortion

$$\left|\frac{dx(t)}{dt}\right|_{max} \le \frac{\Delta}{T_s} \longrightarrow (1)$$

$f_m = 5000\ Hz$

peak amplitude = 1 V

$x(t) = A \cos 2\pi f_m t$ → Sine Sgnl.

$$\frac{dx(t)}{dt} = A \times -(2\pi f_m) \cdot \sin(2\pi f_m t)$$

$$\left|\frac{dx(t)}{dt}\right|_{max} = A \cdot 2\pi f_m \longrightarrow (2)$$

Sub (2) in (1)

$$A\, 2\pi f_m \le \frac{\Delta}{T_s} \Rightarrow \left[\begin{array}{l} \Delta \ge 2\pi A f_m T_s \\ \Delta \ge 2\pi A \dfrac{f_m}{f_s} \end{array}\right]$$

Page No.
Date

gn; $A = 1V$
$f_m = 5000\ Hz$
$f_s = 1000\ Hz$

sub these in eqn (3)

$$\Delta \geq \frac{2\pi A f_m}{f_s} = \frac{2\pi \times 5000}{1000}$$

$$\Delta \geq 10\pi$$

$$\Delta \geq 31.4\ V$$

gn; $f_s = 4 f_N$ $\qquad f_N = 2 f_m$
$= 8 f_m$
$= 8 \times 5000$
$= 40000$

$$\Delta \geq \frac{2\pi \times 1 \times 5000}{40000}$$

$$\Delta \geq \frac{1}{4}\pi$$

$$\Delta \geq 0.7854\ V$$

In case I $\Delta \geq 31.4\ V$ which is larger than sgnl peak value (1V), And it is not practical. It indicate that with $f_s = 1000\ Hz$, you cannot follow a 5000 Hz component without severe slope overload.

Page No.
Date / /

3) A ΔM with a fixed stepsize 0.75 V is given a Sinesoidal msg sgnl, if the sampling freq is 30 times the nyquist rate determine the max permisible amplitude of msg sgnl if slope overload is to be avoided.

To avoid stp Overload distortion - $\left|\frac{dx(t)}{dt}\right|_{max} \leq \frac{\Delta}{T_s} \rightarrow (1)$

$x(t) = A\cos(2\pi f_m t)$

$\left|\frac{dx(t)}{dt}\right|_{max} = A\, 2\pi f_m \rightarrow (2)$

Sub (2) in (1) $\Rightarrow$ $A\, 2\pi f_m \leq \frac{\Delta}{f_s}$ | $A \leq \frac{\Delta f_s}{2\pi f_m}$

gn ; $\Delta = 0.75$ V

$f_s = 30 f_N$

$= 30 \times 2 f_m$

$= 60 f_m$

$A \leq \frac{\Delta f_s}{2\pi f_m} = \frac{0.75 \times 60 \cancel{f_m}}{2\pi \cancel{f_m}}$

$A \leq 7.16$ V

Therefore max permisible amplitude $A_{max} = 7.16$ V.

11.27. DELTA MODULATION

The delta modulator, often denoted Δ modulator, is a process that embeds a low resolution A-to-D converter in a sampled data feedback loop that operates at rates far in excess of the signal's required Nyquist rate.

The motivation for using Delta modulation technique is our awareness that in the conversion process, speed is less expensive than precision, and that by being clever one can use faster signal processing to obtain higher precision.

(i) Reason to use Delta Modulation

We have observed in PCM that it transmits all the bits which are used to code a sample. Hence, signaling rate and transmission channel bandwidth are quite large in PCM. To overcome this problem, Delta Modulation is used.

(ii) Working Principle

Delta modulation transmits only one bit per sample. Here, the present sample value is compared with the previous sample value and this result whether the amplitude is increased or decreased is transmitted. Input signal x(t) is approximated to step signal by the delta modulator. This step size is kept fixed. The difference between the input signal x(t) and staircase approximated signal is confined to two levels, i.e., $+\Delta$ and $-\Delta$. Now, if the difference is positive, then approximated signal is increased by one step, i.e., 'Δ'. If the difference is negative, then approximated signal is reduced by 'Δ'.

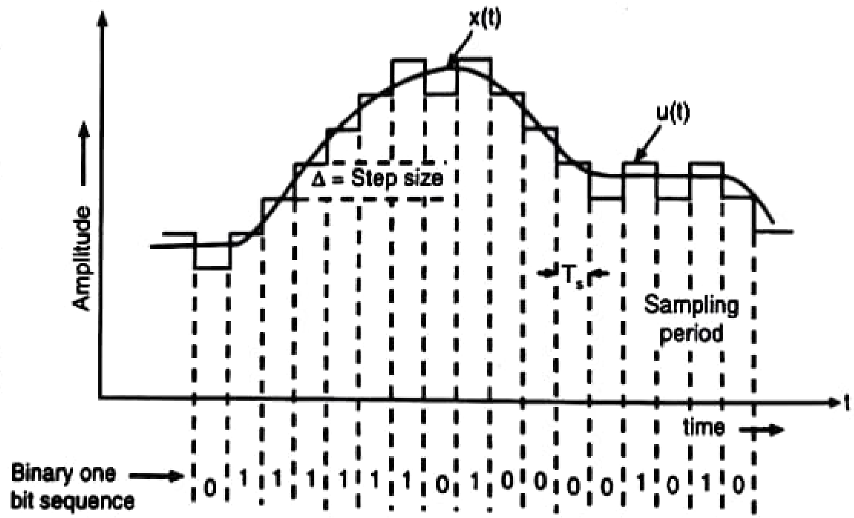


Fig. 11.27. Delta modulation waveform

When the step is reduced, '0' is transmitted and if the step is increased, '1' is transmitted. Hence, for each sample, only one binary bit is transmitted. Figure 11.27 shows the analog signal x(t) and its staircase approximated signal by the delta modulator.

(iii) Mathematical Expressions

Thus, the principle of delta modulation can be explained with the help of few equations as under:

The error between the sampled value of x(t) and last approximated sample is given as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \qquad ...(11.55)$$

where $e(nT_s)$ = error at present sample

$x(nT_s)$ = sampled signal of x(t)

$\hat{x}(nT_s)$ = last sample approximation of the staircase waveform.

If we assume $u(nT_s)$ as the present sample approximation of staircase output,

then, $$u[(n-1)T_s] = \hat{x}(nT_s) \qquad ...(11.56)$$

= last sample approximation of staircase waveform

Let us define a quantity $b(nT_s)$ in such a way that,

$$b(nT_s) = \Delta \text{ sgn } [e(nT_s)] \qquad ...(11.57)$$

This means that depending on the sign of error $e(nT_s)$, the sign of step size Δ is decided. In other words, we can write

$$b(nT_s) = \begin{cases} +\Delta & \text{if } x(nT_s) \geq \hat{x}(nT_s) \\ -\Delta & \text{if } x(nT_s) < \hat{x}(nT_s) \end{cases} \quad ...(11.58)$$

Also, If $b(nT_s) = +\Delta$ then a binary '1' is transmitted

and if $b(nT_s) = -\Delta$ then a binary '0' is transmitted.

Here, T_s = Sampling interval.

(iv) Transmitter Part

Figure 11.28 (a) shows the transmitter (i.e., generation of Delta Modulated signal).

The summer in the accumulator adds quantizer output ($\pm \Delta$) with the previous sample approximation. This gives present sample approximation. i.e.,

$$u(nT_s) = u(nT_s - T_s) + [\pm \Delta]$$

or
$$u(nT_s) = u[(n-1)T_s] + b(nT_s) \quad ...(11.59)$$

The previous sample approximation $u[(n-1)T_s]$ is restored by delaying one sample period T_s. The sampled input signal $x(nT_s)$ and staircase approximated signal $\hat{x}(nT_s)$ are subtracted to get error signal $e(nT_s)$.

DO YOU KNOW?

Delta modulation transmits only one bit per sample, indicating whether the signal level is increacing or decreasing, but it needs a higher sampling rate than PCM for equivalent results.

Thus, depending on the sign of $e(nT_s)$, one bit quantizer generates an output of $+\Delta$ or $-\Delta$. If the step size is $+\Delta$, then binary '1' is transmitted and if it is $-\Delta$, then binary '0' is transmitted.

(v) Receiver Part

At the receiver end, shown in figure 11.28 (b), the accumulator and low-pass filter (LPF) are used. The accumulator generates the staircase approximated signal output and is delayed by one sampling period T_s. It is then added to the input signal. If input is binary '1' then it adds $+\Delta$ step to the previous output (which is delayed). If input is binary '0' then one step 'Δ' is subtracted from the delayed signal. Also, the low-pass filter has the cutoff frequency equal to highest frequency in x(t). This low-pass filter smoothens the staircase signal to reconstruct orignal message signal x(t).

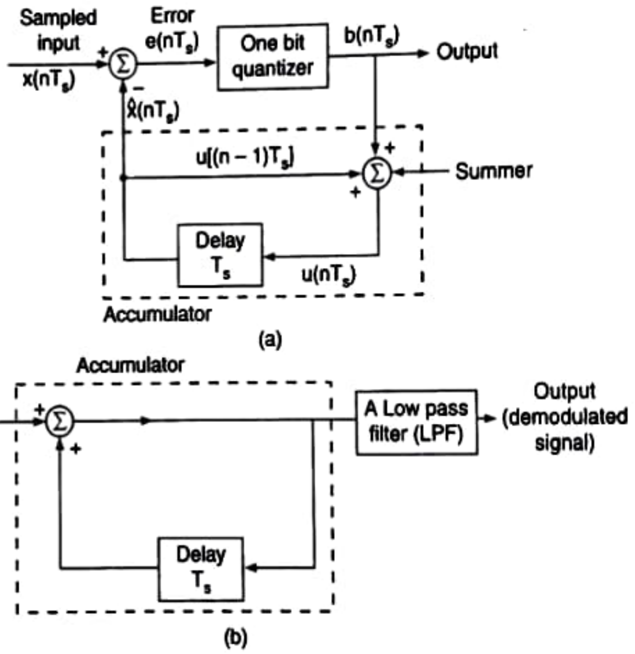


Fig. 11.28. (a) A Delta modulation transmitter (b) A Delta modulation receiver

11.27.1. Advantages of Delta Modulation: Sallent Features of Delta Modulation*

The delta modulation has certain advantages over PCM as under:

(i) Since, the delta modulation transmits only one bit for one sample, therefore the signaling rate and transmission channel bandwidth is quite small for delta modulation compared to PCM.

(ii) The transmitter and receiver implementation is very much simple for delta modulation. There is no analog to digital converter required in delta modulation.

11.27.2. Drawbacks of Delta Modulation**

The delta modulation has two major drawbacks as under:

(i) Slope overload distortion,

(ii) Granular or idle noise

Now, let us discuss these two drawbacks in detail.

* As a matter of fact, digital signals come from a variety of sources. Some sources such as computers are inherently digital. Some sources are analog, but are converted into digital form by a variety of techniques such as PCM and delta modulation (DM).

** Since signal transmission inevitably is subjected to channel noise, such noise will be integrated and will accumulate at the receiver output, which is a highly undesirable phenomenon that is a major drawback of DM.

(i) Slope Overload Distortion

This distortion arises because of large dynamic range of the input signal.

As can be observed from figure 11.29, the rate of rise of input signal x(t) is so high that the staircase signal cannot approximate it, the step size 'Δ' becomes too small for staircase signal u(t) to follow the step segment of x(t). Hence, there is a large error between the staircase approximated signal and the original input signal x(t). This error or noise is known as **slope overload distortion.** To reduce this error, the step size must be increased when slope of signal x(t) is high. Since the step size of delta modulator remains fixed, its maximum or minimum slopes occur along straight lines. Therefore, this modulator is also known as **Linear Delta Modulator (LDM).**

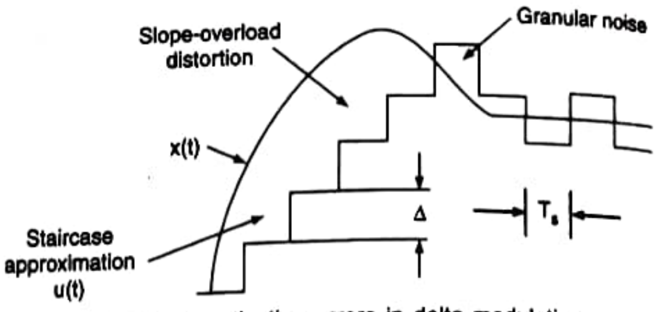


Fig. 11.29. Quantization errors in delta modulation

(ii) Granular or Idle Noise

Granular or Idle noise occurs when the step size is too large compared to small variations in the input signal. This means that for very small variations in the input signal, the staircase signal is changed by large amount (Δ) because of large step size. Figure 11.29 shows that when the input signal is almost flat, the staircase signal u(t) keeps on oscillating by $\pm\,\Delta$ around the signal. The error between the input and approximated signal is called **granular noise.** The solution to this problem is to make step size small.

Important Point: Therefore, a large step size is required to accommodate wide dynamic range of the input signal (to reduce slope overload distortion) and small steps are required to reduce granular noise. Infact, Adaptive delta modulation is the modification to overcome these errors.

11.27.3. Bit Rate (i.e., Signaling Rate) of Delta Modulation*

Delta modulation bit rate (r) = Number of bits transmitted/second

= Number of samples/sec × Number of bits/sample = $f_s \times 1 = f_s$

Key Point: Therefore, the delta modulation bit rate is (1/N) times the bit rate of a PCM system, where N is the number of bits per transmitted PCM codeword. Hence, we can say that the channel bandwidth for the Delta modulation system is reduced to a great extent as compared to that for the PCM system.

Illustrative Examples on Delta Modulation

EXAMPLE 11.13. Given a sine wave of frequency f_m and amplitude A_m applied to a delta modulator having step size Δ. Show that the slope overload distortion will occur if

$$A_m > \frac{\Delta}{2\pi f_m T_s}$$

here T_s is the sampling period.

Solution: Let us consider that the sine wave is represented as,

$$x(t) = A_m \sin(2\pi f_m t)$$

It may be noted that the slope of x(t) will be maximum when derivative of x(t) with respect to 't' will be maximum. The maximum slope of delta modulator may be given as,

$$\text{Maximum slope} = \frac{\text{Step size}}{\text{Sampling period}} = \frac{\Delta}{T_s} \quad ...(i)$$

We know that slope overload distortion will take place if slope of sine wave is greater than slope of delta modulator i.e.,

$$\max\left|\frac{d}{dt}x(t)\right| > \frac{\Delta}{T_s}$$

* Delta modulation, in comparison to PCM (and DPCM), it is a very simple and inexpensive method of A/D conversion. A 1-bit codeword in DM makes word framing unnecessary at the transmitter and the receiver. In fact, this strategy allows us to use fewer bits per sample for encoding a baseband signal.

or $$\max\left|\frac{d}{dt}A_m \sin(2\pi f_m t)\right| > \frac{\Delta}{T_s}$$

or $$\max|A_m 2\pi f_m \cos(2\pi f_m t)| > \frac{\Delta}{T_s}$$

or $$A_m 2\pi f_m > \frac{\Delta}{T_s}$$

or $$A_m > \frac{\Delta}{2\pi f_m T_s}$$ **Hence Proved.**

AMPLE 11.14. A delta modulator system is designed to operate at five times the Nyquist rate for a signal having a ndwidth equal to 3 kHz bandwidth. Calculate the maximum amplitude of a 2 kHz input sinusoidal signal for ich the delta modulator does not have slope over load. Given that the quantizing step size is 250 mV. Also, derive e formula that you use.

lution: In last example, we have derived the relation for slope overload distortion which will occur if,

$$A_m > \frac{\Delta}{2\pi f_m T_s} \quad \text{...(i)}$$

Thus, slope overload will not occur if,

$$A_m \leq \frac{\Delta}{2\pi f_m T_s} \quad \text{...(ii)}$$

The maximum frequency in the signal is,

$$f_m = 3 \text{ kHz}$$

Hence, Nyquist rate $= 2 f_m = 2 \times 3 \text{ kHz} = 6 \text{ kHz}$

Sampling frequency $= 5$ times Nyquist rate

or $f_s = 5 \times 6 \text{ kHz} = 30 \text{ kHz}$

Hence, sampling interval,

$$T_s = \frac{1}{f_s} = \frac{1}{30 \times 10^3} \text{ seconds.}$$

Given that step size $\Delta = 250 \text{ mV} = 250 \times 10^{-3} \text{ V} = 0.25 \text{ V}$

Again, Given that $f_m = 2 \text{ kHz} = 2 \times 10^3 \text{ Hz}$

Substituting all these values in equation (ii), we get

$$A_m \leq \frac{0.25}{2\pi \times 2 \times 10^3 \times \dfrac{1}{30 \times 10^3}}$$

Simplifying, we obtain $A_m \leq 0.6$ volts **Ans.**